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Some Methods of Problem Solving in Historical Mathematical Textbooks

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Abstract

This paper makes an excursion into the history of mathematics as presented in mathematics textbooks. We describe some components of mathematical notions in textbooks by Jakub Kresa (1648-1715) as well as some approaches to the problem solving in textbooks by Franz Močnik (1814-1892). These historical approaches are connected to modern mathematics education because many international studies, such as The Programme for International Student Assessment (PISA) support the use of problem solving and real-life problems in mathematics education. Word tasks have an important place in school mathematics. There are three important stages for solving these tasks: mathematization, calculation and interpretation of the tasks' results. We also highlight divergent thinking in problem solving during the educational process.

Key words: Historical mathematical textbook, History of education, Jakub Kresa, Franz Močnik, Mathematics, Method of generating problems, Problem solving.

Resumo

Este artigo faz um percurso na história da matemática apresentada em livros de matemática. Descrevemos alguns componentes de noções matemáticas em livros didáticos de autoria de Jakub Kresa (1648-1715), bem como algumas abordagens para a resolução de problemas em livros didáticos de autoria de Franz Močnik (1814-1892). Essas abordagens históricas se conectam à educação matemática moderna porque muitos estudos internacionais, como o Programa de Avaliação Internacional de Estudantes (PISA), apoiam o uso de resolução de problemas e de problemas da vida real na educação matemática. Tarefas com palavras têm um lugar importante na matemática escolar. Existem três etapas importantes para resolver essas tarefas: matematização, cálculo e interpretação dos resultados das tarefas. Também destacamos o pensamento divergente na resolução de problemas durante o processo educacional.

Palavras-chave: Livro didático histórico de Matemática, História da educação, Jakub Kresa, Franz Močnik, Método de geração de problemas, Resolução de problemas.

Resumen

Este artículo hace un recorrido por la historia de las matemáticas presentada en los libros de matemáticas. Describimos algunos componentes de las nociones matemáticas en los libros didácticos escritos por Jakub Kresa (1648-1715), así como algunos enfoques para la resolución de problemas en los libros didácticos escritos por Franz Močnik (1814-1892). Estos enfoques históricos se conectan con la educación matemática moderna porque muchos estudios internacionales, como el Programa Internacional de Evaluación de Estudiantes (PISA), apoyan el uso de la resolución de problemas y los problemas de la vida real en la educación matemática. Las tareas con palabras tienen un lugar importante en las matemáticas escolares. Hay tres pasos importantes para resolver estas tareas: matematización, cálculo e interpretación de los resultados de las tareas. También destacamos el pensamiento divergente en la resolución de problemas durante el proceso educativo.

Palabras clave: Libro didáctico histórico de Matemáticas, Historia de la educación, Jakub Kresa, Franz Močnik, Método de generación de problemas, Resolución de problemas.

Introduction

According to Kántor (2013), while it is important in mathematics education to understand mathematical concepts, theorems and proofs, it is more important to understand where they come from and why they are studied. In this regard, it is possible to find such origins and justifications in historical mathematical textbooks or in mathematical books from known mathematicians in the past.

The method of generating problems (see Wittman, 2001)) is often used in historical mathematical textbooks. In this method, pupils consider their own problems and can ask for help when necessary. After the first problem has been completely solved and clarified, the teacher and students can together think about further questions and generate problems which are related to the solved problem. Thus the original (first) problem acts as a generator problem (GP). Related problems are obtained by analogy, variation, generalization, specialization and so on. The group of all new problems, together with their GP, is called the set of generated problems of the GP or the problem domain of the GP.

The idea of generating new problems related to a given problem is also often used in the context of a specific educational focus – namely, fostering a child's mathematical creativity. As described in Singer et al. (2016), the research team of Mihaela Singer took the approach of problem posing to foster so-called "cognitive flexibility" (which is described by cognitive variety, cognitive novelty and changes in cognitive framing) by the method of problem posing. Such cognitive variety "manifests in the formulation of different new problems/properties from an input stimulus" (Singer et al., p. 15). Singer et al. (2013) explain further that the "problem-posing research is an emerging force within mathematics education, which offers a variety of contexts for studying and developing abilities in mathematically promising students" (Singer et al., 2016, p. 18).

Another major theory in mathematics education, especially with regard to fostering mathematical giftedness and creativity, is the idea of using multiple-solution tasks (MSTs), as suggested by Leikin (2009, 2013), for example, where MSTs can be used as a lens to observe creativity in a problem-solving context.

In the context of historical approaches, reviewing authentic works from the history of mathematics can help bring them "back to life". As mentioned in Klassen (2006), the "humanizing and clarifying influence of history of science brings the science to life and enables the student to construct relationships that would have been impossible in the traditional decontextualized manner in which science has been taught" (p. 48). Whereas a direct use of original historical artefacts may be problematic, Kubli (2002, 1999) reports that students react more positively to historical materials when they are prepared in a narrative form. Bruner (1986) opposes the "logical-scientific mode" in favour of the "narrative mode", too. Emotionally engaging learners by applying a narrative didactic approach in mathematics education is, for example, intended by Brandl (2016).

Jakub Kresa and his Textbook: *Analysis Speciose Trigonometriae Sphaericae*

Jakub Kresa (1648–1715) was a Czech Jesuit mathematician who taught mathematics in Spain for a long time. The textbook *Analysis speciose trigonometriae sphaericae* prepared by Jakub Kresa was intended for sailors, most of whom had not studied mathematics in schools. For this reason, the whole name of the textbook is much longer: *Analysis speciose trigonometriae sphaericae Primo mobili Triangulis Rectilineis Progressioni Arithmeticae et Geometricae Alisque variis problematis aplicata*. The translated title is *Special analysis of spherical trigonometry first of all of plane triangles with of arithmetical and geometrical progression with a variety of applied problems*. Spherical geometry is addressed in the last part of the book.

The majority of the textbook is devoted to the basic arithmetic and concepts of plane geometry, which is needed for understanding spherical geometry, which helps when working with a sextant on the sea. In Kresa's age, sailors used the location of the sun during the day or of a star (mostly the polar star) during the night to find the place of their ship on the sea. Jakub Kresa spent much time in Spain, where the navigation used for ship transportation was important. A sextant was a good tool for finding the location of a ship on the sea (see Figure 1).

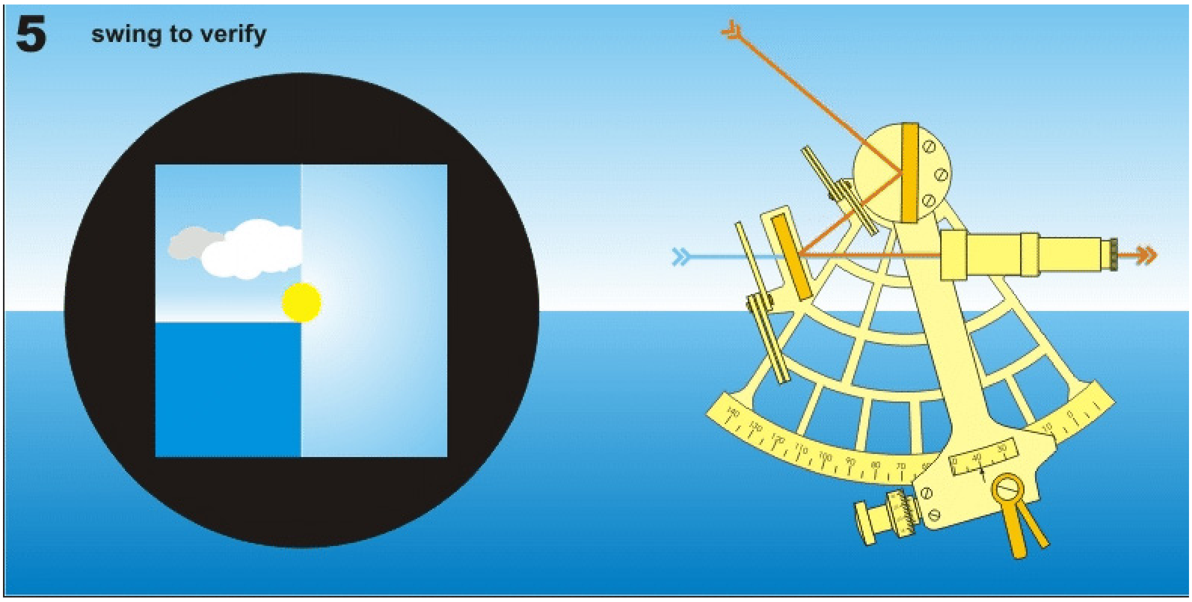

Figure 1. Using a sextant to find the angle of the sun (from www.wikipedia.org)

In this textbook, Jakub Kresa shows his own method of problem solving in the following task. We here translate the formulation and solutions of the task from the Latin.

Titus defines in his testament: The first child from his children receives 300 and one sixteenth of the rest. After that, the second child receives 600 and one sixteenth of the rest. This algorithm is possible to repeat for any other children. Every child gets the same part of heritage. How big is the heritage, and how many children receive the same part of the heritage?

AD ANALYSIM SPECIOSAM. 109

Problema IV. Hæreditas ſine rixa.

Titius mandavit in ſuo Teſtamento: ut Primogenitus ex Aſſe acciperet primo 300. & dein reſidui ſextam partem. Secundogenitus primò acciperet (ex Reſiduo) 600, & ex hinc reſiduo $\frac{1}{6}$. Et ità reliqui hæredes, ut ſequens ſemper primò acciperet 300. ampliùs, quàm antecedens, & dein reſidui ſextam partem. Factâ repartitione, inventæ ſunt omnium portiones æquales. Quæritur: quanta fuerit hæreditas? & numerus hæredum?

Figure 2. The task about heritage in Latin (see Kresa, 1720)

He presents two solutions to this problem:

Solution I. Let the heritage = *y* and *a* = 300. So, the first child receives *a* and the rest is *y* - *a*. Now one sixteenth of the rest is $\frac{y-a}{6}$. Because *a* = $\frac{6a}{6}$, the sum for first child is = $\frac{y+5a}{6}$. For other children, the rest of heritage we receive if we subtract the sum for first child from the heritage. Because y is $\frac{6y}{6}$, the rest of the heritage is $\frac{5y-5a}{6}$. Now the second child receives 2a, which is $\frac{12a}{6}$.

For him, the rest = $\frac{5y-17a}{6}$, which means $\frac{1}{6}=\frac{5y-17a}{36}$. The whole part of heritage for the second child = 2*a* + $\frac{5y-17a}{36}$. Because the parts of both children are equal: $\frac{y+5a}{6}$ = 2*a* + $\frac{5y-17a}{36}$. When we multiply by 36, then 6*y* + 30*a* = 72*a* + 5*y* - 17*a*, and this is 55*a* + 5*y*. We obtain *y* = 25*a* = 7500. This means that the first child obtains 1500, all children 7500. Hence the number of children = 5 = the number of children with some remaining heritage.

ANALYSIS.

Sit hæreditas $= y$. & $300 = a$. Ergo, ubi primus accepit a. reſiduum $= y - a$. Cujus $\frac{1}{6} = \frac{y-a}{6}$, cui ſi addatur a, ſeu $\frac{6a}{6}$, erit ſumma $= \frac{y+5a}{6}$. Quæ eſt portio Primogeniti; quâ ſubtractâ ex y, ſeu ex $\frac{6y}{6}$, erit reſiduum $= \frac{5y-5a}{6}$.

Ex hoc reſiduo ſecundus accipit $2a$, ſeu $\frac{12a}{6}$, ergo manebit

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110 LIBER I. ISAGOGE

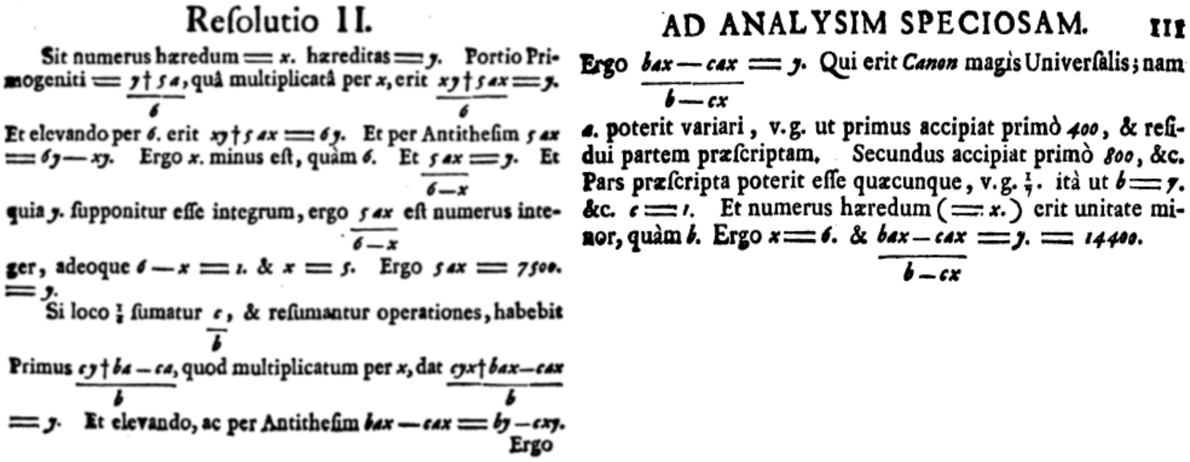
nebit reſiduum $= \frac{5y-17a}{6}$, cujus $\frac{1}{6} = \frac{5y-17a}{36}$. Adeoque portio Secundo-geniti $= 2a + \frac{5y-17a}{36}$. Et cùm portiones dentur æquales, ergo $\frac{y+5a}{6} = 2a + \frac{5y-17a}{36}$.

Et elevando per 36. erit $6y + 30a = 72a + 5y - 17a$, ſeu $55a + 5y$. Et per Antitheſim: $y = 25a = 7500$.

Ergo portio Primo-geniti $= 1500$, per quam diviſo aſſe $= 7500$, erit quotus $= 5 =$ numero hæredum.

Figure 3. Solution I in the original Latin (see Kresa, 1720)

Solution II. Let x be the number of children with some rest heritage. The heritage = y. The sum for the first child = $\frac{y+5a}{6}$. If we multiply this term by x, we receive $\frac{xy+5ax}{6}$ = y. If we multiply by 6, we receive $xy + 5ax = 6y$. If we subtract $5ax = 6y - xy$, the number x is less than 6 and $\frac{5ax}{6-x}$ = y. The number y must be a whole number; for this reason, $6 - x = 1$ and $x = 5$. Hence $5ax = 7500$. If instead of $\frac{1}{6}$ we use $\frac{c}{b}$ and we repeat the operations, we receive for the first child $\frac{cy+ba-ca}{b}$. If we multiply this with x, we receive $\frac{cyx+bax-cax}{b}$ = y. If we multiply and we make subtraction $bax - cax = by - cxy$. Hence, $\frac{bax-cax}{b-cx}$ = y. We receive a universal formula with parameter a. So if we now change the condition of the task, that the first child receives 400 and $\frac{1}{7}$ of the rest, the second child receives 800 and $\frac{1}{7}$ of the rest and so on, we can give $b = 7$ & $c = 1$. The number of children is less than b. Hence, $x = 6$ and $\frac{bax-cax}{b-cx}$ = y = 14400 (in the expression, we give $a = 400$, $x = 6$, $b = 7$, $c = 1$).



Resolutio II.

Sit numerus hæredum $= x$. hæreditas $= y$. Portio Primogeniti $= \frac{y+5a}{6}$, quâ multiplicatâ per x, erit $\frac{xy+5ax}{6} = y$.
Et elevando per 6. erit $xy + 5ax = 6y$. Et per Antithesim $5ax = 6y - xy$. Ergo x. minus est, quàm 6. Et $\frac{5ax}{6-x} = y$. Et quia y. supponitur esse integrum, ergo $\frac{5ax}{6-x}$ est numerus integer, adeoque $6 - x = 1$. & $x = 5$. Ergo $5ax = 7500. = y$.
Si loco $\frac{1}{6}$ sumatur $\frac{c}{b}$, & resumantur operationes, habebit Primus $\frac{cy+ba-ca}{b}$, quod multiplicatum per x, dat $\frac{cyx+bax-cax}{b} = y$. Et elevando, ac per Antithesim $bax - cax = by - cxy$. Ergo

AD ANALYSIM SPECIOSAM. 111

Ergo $\frac{bax-cax}{b-cx} = y$. Qui erit *Canon* magis Universalis; nam a. poterit variari, v.g. ut primus accipiat primò 400, & residui partem præscriptam. Secundus accipiat primò 800, &c. Pars præscripta poterit esse quæcunque, v.g. $\frac{1}{7}$. ità ut $b = 7$. &c. $c = 1$. Et numerus hæredum ($= x$.) erit unitate minor, quàm b. Ergo $x = 6$. & $\frac{bax-cax}{b-cx} = y. = 14400$.

Figure 4. Solution II in the original Latin

It was typical at that time for every task to be presented with more than one solution, which can be a good approach for modern teaching. In this case, it was possible to show that the second solution generalizes the task, which is helpful for the understanding of pupils – that is, presenting problem solving for pupils in multiple ways. Jakub Kresa was an author in 17th century, and it will be described in the next part some aspects of mathematics education in the beginning of the 19th century, during which the institutional form of teacher training was created in central Europe.

Personality of Franc Močnik (1814-1892)

There was in the year 2014 the 200th anniversary of the birth of the significant personality in the field of principal mathematical textbooks of the Austro-Hungarian monarchy in the second half of the 19th century, Dr Franc Močnik.

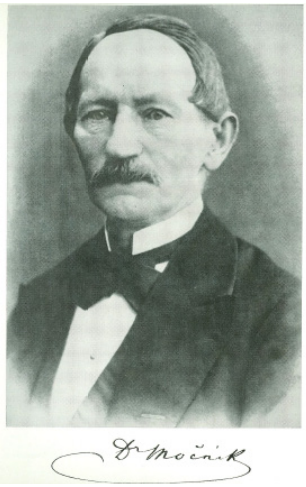
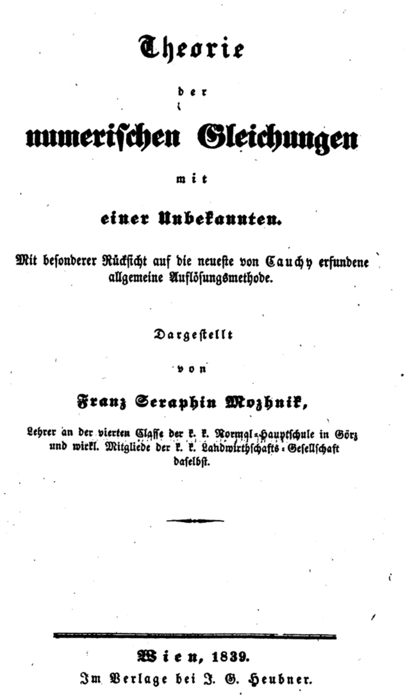


Figure 5. Franc Močnik (1 October 1814 Cerkno – 3 November 1892 Graz) (see Povsic, 1966)

Franc Močnik was born on 1 October 1814 in Cerkno, Slovenia, as a son of Slovenian peasants, Andrej and Marjana Močniks (see Povsic, 1966). He attended the primary school in the town of Idrija and the secondary grammar school in Ljubljana. Later, he studied at the Faculty of Theology in Gorica (today, a town on the Italian-Slovenian border), but he did not become a priest. From 1836 to 1846, he worked as a teacher at the normal school in Gorica.



Theorie
der
numerischen Gleichungen
mit
einer Unbekannten.
Mit besonderer Rücksicht auf die neueste von Cauchy erfundene allgemeine Auflösungsmethode.
Dargestellt
von
Franz Seraphin Mozhnik,
Lehrer an der vierten Classe der k. k. Normal-Hauptschule in Görz und wirkl. Mitglied der k. k. Landwirthschafts-Gesellschaft daselbst.

Wien, 1839.
Im Verlage bei J. G. Heubner.

Figure 6. The book on Cauchy's method by Franc Močnik

At that time, he met the French mathematician Augustin Louis Cauchy (1787–1857), who was living in the area for political reasons. Močnik presented his method of numerical solutions of equations in German language in his work *Theorie der numerischen Gleichungen mit einer Unbekannten,* which was published in 1839. Meeting Cauchy motivated Močnik to study mathematics at the university in Graz, from which he graduated in 1840 with a doctorate in philosophy.

In 1846, Močnik became a professor of elementary mathematics at the Technical Academy in Lviv (modern-day Ukraine). From 1849 to 1851, he was a professor of mathematics at the University in Olomouc (modern-day Czech Republic, see Jeraj, 1995).

From 1851 to 1860, he worked as an inspector of primary schools in Ljubljana. In 1861, he was appointed as inspector of primary and real schools (Realschule) in Graz for Styria and Carinthia. In 1869, Močnik became a provincial inspector for the province Styria. He retired for medical reasons in 1871. While still actively writing, he lived in Graz until he passed away, on 30 November 1892.

Močnik wrote many mathematical textbooks. According to Branko Sustar (see Sustar, 2014), his last bibliography was compiled by Jose Povsic. Močnik's textbooks were originally published in German (148 textbooks in 980 editions), and they were translated into 14 other languages: 39 Slovenian textbooks (174 editions), 29 Croatian textbooks (132 editions), 32 Serbian textbooks (77 editions), 4 textbooks for Bosnia and Herzegovina (36 editions), 9 Albanian textbooks (13 editions), 9 Bulgarian textbooks (23 editions), 39 Czech textbooks (109 editions), 46 Italian textbooks (130 editions), 38 Hungarian textbooks (185 editions), 4 Greek textbooks (4 editions), 39 Polish textbooks (86 editions), 20 Romanian textbooks (36 editions), 5 Slovak textbooks (5 editions) and 40 Ukrainian textbooks (74 editions).

Franz Močnik formulated many examples from real life. Below, we present some examples from his book “Teaching counting” (see Figure 7 and Močnik, 1871).

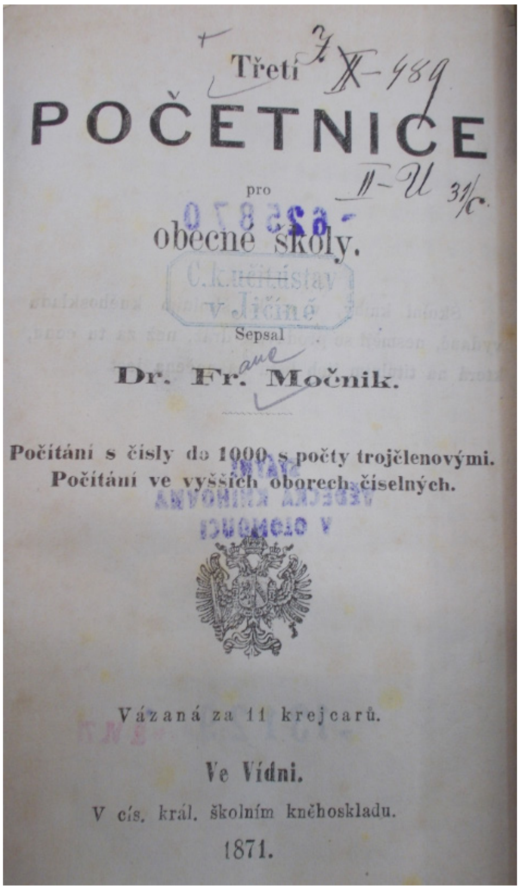

Třetí
POČETNICE
pro
obecné školy.
Sepsal
Dr. Fr. Močnik.
Počítání s čísly do 1000 s počty trojčlenovými.
Počítání ve vyšších oborech číselných.
Vázaná za 11 krejcarů.
Ve Vídni.
V cís. král. školním knihoskladu.
1871.

Figure 7. The textbook “Counting teaching” (Početnice)

Now, two examples from this Czech textbook (Counting teaching) will be presented in a free translation from Czech to English describing the texts of the tasks. The solutions are not from the mentioned textbook; they are some of them which are useful for teaching.

On page 19 is following task:

97. The farmer had from his fields 215 hectolitres of wheat, 306 hectolitres of rye and 127 hectolitres of barley. He sold from this 168 hectolitres of wheat, 135 hectolitres of rye and 48 hectolitres of barley. How many hectolitres did he spare from every kind of corn? How many hectolitres did he spare from all the corn together?

Solution: This task is suitable for pupils at the primary level. It is important that the pupils read the text of the task carefully. The farmer will have as the rest 215 – 168 = 47 hectolitres of wheat, 306 – 135 = 171 hectolitres of rye and 127 – 48 = 79 hectolitres of barley. This means that the farmer has as a remainder 47 + 171 + 79 = 297 hectolitres of corn.

It is possible to give the pupils two other additional questions. Firstly, how many hectolitres of corn did the farmer sell? It is possible to calculate that it was 168 + 135 + 48 = 351 hectolitres of corn.

Secondly, how many hectolitres of corn did the farmer have from his fields? It is 215 + 306 + 127 = 648 hectolitres of corn. If the farmer sold from 648 hectolitres of corn 351 hectolitres, then the rest is 648 – 351 = 297 hectolitres. This fact shows that this task has a number of correct solutions. It is a very important educational tool for supporting pupils' understanding of divergent meaning.

On the page 70 is the following task:

94. How much less will the area of the floor be of some room in the family house if the floor has a length of 921 centimetres and a width of 755 centimetres and if we nail to the perimeter of the floor a strip which has a width of 4 centimetres (we can expect that the floor is an rectangle)?

Solution: The area of the room is 921 times 755 square centimetres. It is 695,355 square centimetres.

If we have the floor with the strip, then we have the rest, which has a length of 921 – 4 = 917 centimetres and a width of 755 – 4 = 751 centimetres. That means that the area of the rest is 917 times 751 centimetres, which is 688,687 square centimetres. This means that we lost 695,355 – 688,687 = 6,668 square centimetres.

Conclusion

Modern teaching of mathematics based on historical mathematical textbooks is relevant because the methodological framework of the curriculum is based on the models and tasks from the cultural environment of the children and their parents. These textbooks can help both current and future teachers of mathematics. A great number of the materials in these textbooks can be used in the modern e-learning courses by the use of appropriate educational software (see Koreňová, 2016;, Kopáčová-Žilková, 2015).

In his task "Heritage without conflict", Jakub Kresa (1648-1715) solved a concrete example, which is still useful for teaching in the field of mathematics with regard to word tasks, the development of algebraic thinking and interpretation the calculated results.

It is important for mathematical education and for math teacher training programs that students develop their ability to explain every problem in multiple ways; in this regard, they need to find the solution using methods that differ from that of their teacher. In their works, Jakub Kresa and Franz Močnik solved mostly practical problems and tried to explain their findings to people who had not formally studied mathematics.

It is possible in the future that many works of mathematics by different historical authors can be presented in modern form through ICT tools and educational software, because a lot of original historical mathematical works and textbooks is possible to find in electronic form in internet or in electronical libraries of archives (see also Partová, 2002). These activities can help in popularisation and teaching of mathematics or science education in every kind of schools (see Vancova, Sulovska, 2016).

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